Chapter 12

Cross-Layer Optimization for Multi-Hop Cognitive Radio Networks
Outline

- CR network (CRN) properties
- Mathematical models at multiple layers
- Case study
Traditional Radio vs CR

Traditional radio

- Hardware-based, not flexible
- Operates on a given frequency band, inefficient use of spectrum

CR - A revolution in radio technology

- Software-based, most functions are programmable
- Can sense available spectrum
- Can switch to and operate on different frequency bands
CRN Properties

- In CRN, the set of available bands may be different at different nodes.
  - Two nodes can communicate only if they have a common available band.
- Each band may have a different bandwidth.
- Each node can use multiple bands at the same time.
Opportunity and Challenge

- CRN properties provide opportunity to achieve better performance
  - Increase spectrum efficiency, larger throughput ...

- CRN properties also bring unique challenge on how to fully exploit CRN capability
  - Previous approaches assume homogeneous band setting, cannot be applied to CRN
Our Approach

- Build mathematical models for multiple layers
  - Also build cross-layer constraint to fully exploit CRN capability
- Formulate an optimization problem
- Apply optimization techniques to solve the problem
Scheduling Constraints

- We perform Scheduling on bands
  - Recall that each node has multiple available bands

\[
x_{ij}^m = \begin{cases} 
1 & \text{If node } i \text{ transmits data to node } j \text{ on band } m, \\
0 & \text{otherwise.}
\end{cases}
\]

- A node \( i \) cannot transmit and receive on the same band

\[
x_k^m + x_{ij}^m \leq 1 \quad (j, k \in T_i^m, j \neq k)
\]

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Scheduling Constraints (cont’d)

- A node $i$ cannot transmit to or receive from multiple nodes on the same band

$$\sum_{j \in T^m_i} x^m_{ij} \leq 1 \quad \text{and} \quad \sum_{k \in T^m_i} x^m_{ki} \leq 1$$

- These scheduling constraints can be combined as

$$\sum_{k \in T^m_i} x^m_{ki} + \sum_{j \in T^m_i} x^m_{ij} \leq 1$$

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Power Control Constraint

- Power control can be done on $Q$ levels between $0$ and $P_{max}$
  - Power level: $q_{i,j}^m \in \{0, 1, 2, \ldots, Q\}$
  - Transmission power is: $\frac{q_{i,j}^m}{Q} P_{max}$

- The cross-layer relationship between scheduling variable and power level is
  $$q_{i,j}^m \leq Q x_{i,j}^m$$
SINR Formula

- SINR at the receiver

\[ s_{ij}^m = \frac{g_{ij} q_{ij}^m P_{\text{max}}}{\eta W + \sum_{k \neq i,j} \sum_{h \neq i,j} g_{kj} q_{kh}^m P_{\text{max}} + \sum_{k \in \mathcal{N}} \sum_{h \in T_k^m} g_{kj} q_{kh}^m} \]

- Let \( t_k^m = \sum_{h \in T_k^m} q_{kh}^m \). We have

\[ s_{ij}^m = \frac{g_{ij} q_{ij}^m}{\eta W Q P_{\text{max}} + \sum_{k \neq i,j} g_{kj} t_k^m} \]

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Physical Interference Model

- A transmission is successful $\Leftrightarrow$ SINR exceeds certain threshold $\alpha$

- The relationship between SINR and scheduling variable is

$$s_{ij}^m \geq \alpha x_{ij}^m$$
Routing Constraints

- For maximum flexibility (and optimality), we allow flow splitting (multi-path routing)

- Flow balance constraints for a session $l$
  - If node $i$ is source, then
    $$\sum_{j \in T_i} f_{ij}(l) = r(l)K$$
  - If node $i$ is destination, then
    $$\sum_{k \in T_i} f_{ki}(l) = r(l)K$$
  - For all other cases, we have
    $$\sum_{j \neq s(l)} f_{ij}(l) = \sum_{k \neq d(l)} f_{ki}(l)$$

Link Capacity Constraint

- The cross-layer relationship between flow rate and SINR
  - Aggregate flow on a link cannot exceed its capacity

\[ \sum_{l \in L} f_{ij}(l) \leq \sum_{m \in \mathcal{M}_{ij}} W \log_2(1 + s_{ij}^m) \]
Outline

- CR network (CRN) properties
- Mathematical models at multiple layers
- Case study
Throughput Maximization Problem

- Consider a multi-hop CRN
- Each session has a minimum rate requirement $r(l)$
- Multi-hop multi-path routing for each session
- Aim to maximize a rate scaling factor $K$ for all sessions
Problem Formulation

- Based on our models at multiple layers, we have

\[
\begin{align*}
\text{Max} & \quad \sum_{i \in T_k^m} x_{ki}^m + \sum_{j \in T_i^m} x_{ij}^m \leq 1 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i) \\
\text{s.t.} & \quad q_{ij}^m - Qx_{ij}^m \leq 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in T_i^m) \\
& \quad \sum_{j \in T_i^m} q_{ij}^m - t_i^m = 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i) \\
& \quad \frac{\eta W_Q}{P_{\text{max}}} s_{ij}^m + \sum_{k \neq i, j}^{k \in \mathcal{N}} g_{kj} t_k^m s_{ij}^m - g_{ij} q_{ij}^m = 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in T_i^m)
\end{align*}
\]
Problem Formulation (cont’d)

\[ \alpha x_{ij}^m - s_{ij}^m \leq 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in T_i^m) \]

\[ \sum_{l \in \mathcal{L}} f_{ij}(l) - \sum_{m \in \mathcal{M}_{ij}} W \log_2 (1 + s_{ij}^m) \leq 0 \quad (i \in \mathcal{N}, j \in T_i) \]

\[ \sum_{j \in T_i} f_{ij}(l) - r(l) K = 0 \quad (l \in \mathcal{L}, i = s(l)) \]

\[ \sum_{j \in T_i} f_{ij}(l) - \sum_{k \neq d(l)} f_{ki}(l) = 0 \quad (l \in \mathcal{L}, i \in \mathcal{N}, i \neq s(l), d(l)) \]

\[ x_{ij}^m \in \{0, 1\}, q_{ij}^m \in \{0, 1, 2, \cdots, Q\}, t_i^m, s_{ij}^m \geq 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in T_i^m) \]

\[ K, f_{ij}(l) \geq 0 \quad (l \in \mathcal{L}, i \in \mathcal{N}, i \neq d(l), j \in T_i, j \neq s(l)) \]

Mixed integer non-linear program
Our Approach
Branch-and-Bound

☐ A form of the *divide-and-conquer* technique
  ■ Divide the original problem into sub-problems
  ■ Find upper and lower bounds for each sub-problem
    ☐ Upper bound obtained via convex hull relaxation
    ☐ Lower bound obtained via a local search algorithm
  ■ We then have the upper and lower bounds for the original problem
    ☐ Once these two bounds are close to each other, we are done
    ☐ Otherwise, we further divide and obtain more sub-problems

☐ The obtained solution is near-optimal

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Branch-and-Bound: An Example

Original Problem 1

Bounds for Scaling Factor

UB = UB₁

LB = LB₁
Branch-and-Bound (cont’d)

Bounds for Scaling Factor

Problem 2

Problem 3

UB = UB

LB = LB

LB = LB

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Branch-and-Bound (cont’d)

![Diagram showing branch-and-bound process with bounds for scaling factor and problems 2, 4, and 5.](image)
Branch-and-Bound

- Divide the original problem into sub-problems
- Find upper and lower bounds for each sub-problem
  - Upper bound obtained via convex hull relaxation
  - Lower bound obtained via a local search algorithm
- We then have the upper and lower bounds for the original problem
  - Once these two bounds are close to each other, we are done
  - Otherwise, we further divide and obtain more sub-problems
Linear Relaxation for Product Terms

Reformulation Linearization Technique (RLT)

For the non-linear term of \( t^m_k s^m_{ij} \), suppose
\[
(t^m_k)_L \leq t^m_k \leq (t^m_k)_U \quad \text{and} \quad (s^m_{ij})_L \leq s^m_{ij} \leq (s^m_{ij})_U
\]
We have
\[
[t^m_k - (t^m_k)_L] \cdot [s^m_{ij} - (s^m_{ij})_L] \geq 0
\]
That is,
\[
(t^m_k)_L \cdot s^m_{ij} + (s^m_{ij})_L \cdot t^m_k - u^m_{ijk} \leq (t^m_k)_L \cdot (s^m_{ij})_L
\]
Other three linear relaxation constraints are
\[
(t^m_k)_U \cdot s^m_{ij} + (s^m_{ij})_L \cdot t^m_k - u^m_{ijk} \geq (t^m_k)_U \cdot (s^m_{ij})_L
\]
\[
(t^m_k)_L \cdot s^m_{ij} + (s^m_{ij})_U \cdot t^m_k - u^m_{ijk} \geq (t^m_k)_L \cdot (s^m_{ij})_U
\]
\[
(t^m_k)_U \cdot s^m_{ij} + (s^m_{ij})_U \cdot t^m_k - u^m_{ijk} \leq (t^m_k)_U \cdot (s^m_{ij})_U
\]
Linear Relaxation for log Terms

Convex hull relaxation

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Relaxed Problem Formulation -- LP

Max \quad K

s.t. \quad \sum_{i \in T_k^m} x_{ki}^m + \sum_{j \in T_i^m} x_{ij}^m \leq 1 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i)

\quad q_{ij}^m - Qx_{ij}^m \leq 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in T_i^m)

\quad \sum_{j \in T_i^m} q_{ij}^m - t_i^m = 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i)

\quad \frac{nWQ}{P_{\text{max}}} s_{ij}^m + \sum_{k \in \mathcal{N}} g_{kj} u_{ijk}^m - g_{ij} q_{ij}^m = 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in T_i^m)

\quad \text{RLT constraints for } u_{ijk}^m \quad (i, k \in \mathcal{N}, m \in \mathcal{M}_i, j \in T_i^m, k \neq i, j)

\quad \alpha x_{ij}^m - s_{ij}^m \leq 0 \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in T_i^m)

\quad \sum_{l \in \mathcal{L}} f_{ij}(l) - \sum_{m \in \mathcal{M}_i} \frac{W}{\ln 2} c_{ij}^m \leq 0 \quad (i \in \mathcal{N}, j \in T_i)

\quad \text{Convex hull constraints for } c_{ij}^m \quad (i \in \mathcal{N}, m \in \mathcal{M}_i, j \in T_i^m)

\quad \sum_{j \in T_i} f_{ij}(l) - r(l) K = 0 \quad (l \in \mathcal{L}, i = s(l))

\quad \sum_{j \neq s(l)} f_{ij}(l) - \sum_{k \neq d(l)} f_{ki}(l) = 0 \quad (l \in \mathcal{L}, i \in \mathcal{N}, i \neq s(l), d(l))

\quad c_{ij}^m, u_{ijk}^m \geq 0 \quad (i, k \in \mathcal{N}, m \in \mathcal{M}_i, j \in T_i^m, k \neq i, j)

\quad K, f_{ij}(l) \geq 0 \quad (l \in \mathcal{L}, i \in \mathcal{N}, i \neq d(l), j \in T_i, j \neq s(l))

\quad (x, q, t, s) \in \Omega_z ,

Can be solved in polynomial time

Provides an upper bound
Branch-and-Bound

- Divide the original problem into sub-problems
- Find upper and lower bounds for each sub-problem
  - Upper bound obtained via convex hull relaxation
  - Lower bound obtained via a local search algorithm
- We then have the upper and lower bounds for the original problem
  - Once these two bounds are close to each other, we are done
  - Otherwise, we further divide and obtain more sub-problems
Obtain Lower Bound

- In the solution to the upper bound, both $x$ and $q$ variables may not be integers
  - Infeasible to the original problem

- Design a local search algorithm to find a feasible solution

- This feasible solution provides a lower bound
Local Search Algorithm

- Begin with the minimum transmission power at each node
- Iteratively increase the bottleneck link’s capacity
  - Compute each link’s capacity
  - Identify the bottleneck link for $K$
  - Try to increase its transmission power on some band -- Ensure all constraints hold
  - Algorithm terminates when the transmission power cannot be increased
Branch-and-Bound

- Divide the original problem into sub-problems
- Find upper and lower bounds for each sub-problem
  - Upper bound obtained via convex hull relaxation
  - Lower bound obtained via a local search algorithm
- We then have the upper and lower bounds for the original problem
  - Once these two bounds are close to each other, we are done
  - Otherwise, we further divide and obtain more sub-problems
Obtain New Problems

- Choose the problem with the largest upper bound
- Divide this problem into two problems
  - Select a partition variable
  - Divide its value interval into two intervals
- Each new problem may have tighter bounds
Select Partition Variable

- Selection of partition variable will affect the bounds to new problems

- To obtain tighter bounds, we first select an $X$ variable
  - Scheduling variables are more significant
  - Among $X$ variables, we choose the one with the maximum relaxation error

- We then select a $q$ variable based on relaxation error
Numerical Results

Network setting
- 20, 30, or 50 nodes in a 50 x 50 area
- 5 or 10 communication sessions with minimum rate requirement in [1, 10]
- 10 bands with the same bandwidth of 50
  - A subset of these 10 bands is available at each node
- 10 power levels
- Aim to find 90% optimal solution
## A 20-Node Network

### Table 12.3: Location and available frequency bands at each node for a 20-node network.

<table>
<thead>
<tr>
<th>Node</th>
<th>Location</th>
<th>Available Bands</th>
<th>Node</th>
<th>Location</th>
<th>Available Bands</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.1, 9.9)</td>
<td>1, 2, 3, 4, 7, 8, 9, 10</td>
<td>11</td>
<td>(28.1, 25.6)</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10</td>
</tr>
<tr>
<td>2</td>
<td>(29.2, 31.7)</td>
<td>1, 2, 3, 4, 5, 7, 8, 10</td>
<td>12</td>
<td>(32.3, 38)</td>
<td>1, 8, 9, 10</td>
</tr>
<tr>
<td>3</td>
<td>(3, 31.1)</td>
<td>1, 4, 5, 6</td>
<td>13</td>
<td>(47.2, 2.6)</td>
<td>3, 5, 10</td>
</tr>
<tr>
<td>4</td>
<td>(11.8, 40.1)</td>
<td>1, 2, 3, 4, 6, 9, 10</td>
<td>14</td>
<td>(44.7, 15)</td>
<td>2, 3, 6, 7, 8</td>
</tr>
<tr>
<td>5</td>
<td>(15.8, 9.7)</td>
<td>1, 2, 3, 5, 6, 8, 9</td>
<td>15</td>
<td>(44.7, 24)</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10</td>
</tr>
<tr>
<td>6</td>
<td>(16.3, 19.5)</td>
<td>3, 5, 6, 8, 9</td>
<td>16</td>
<td>(47.9, 43.8)</td>
<td>1, 3</td>
</tr>
<tr>
<td>7</td>
<td>(0.6, 27.4)</td>
<td>1, 4, 8, 9, 10</td>
<td>17</td>
<td>(46.4, 16.8)</td>
<td>1, 7, 9</td>
</tr>
<tr>
<td>8</td>
<td>(22.6, 40.9)</td>
<td>1, 2, 3, 5, 7, 9, 10</td>
<td>18</td>
<td>(11.5, 12.2)</td>
<td>2, 5, 6, 10</td>
</tr>
<tr>
<td>9</td>
<td>(35.3, 10.3)</td>
<td>2, 9</td>
<td>19</td>
<td>(28.2, 14.8)</td>
<td>4, 5, 6, 7, 8, 9, 10</td>
</tr>
<tr>
<td>10</td>
<td>(31.9, 19.6)</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9, 10</td>
<td>20</td>
<td>(2.5, 14.5)</td>
<td>1, 7, 10</td>
</tr>
</tbody>
</table>

### Table 12.4: Source node, destination node, and minimum rate requirement of each session in the 20-node network.

<table>
<thead>
<tr>
<th>Session $l$</th>
<th>Source Node $s(l)$</th>
<th>Dest. Node $d(l)$</th>
<th>Min. Rate $r(l)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>17</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: The last two rows indicate an error in the table, as they do not correspond to any of the previous entries.
A 20-Node Network Topology
The 20-Node Network: Transmission Powers

Band 1: $q_{7,3}^1 = 1$, $q_{16,12}^1 = 7$;
Band 2: $q_{8,2}^2 = 2$;
Band 3: $q_{13,14}^3 = 2$;
Band 4: $q_{1,7}^4 = 7$, $q_{2,10}^4 = 2$;
Band 5: $q_{11,10}^5 = 1$;
Band 6: $q_{15,19}^6 = 9$;
Band 7: $q_{14,17}^7 = 1$, $q_{20,1}^7 = 1$;
Band 8: $q_{12,11}^8 = 3$;
Band 9: $q_{12,8}^9 = 1$, $q_{19,6}^9 = 3$;
Band 10: $q_{18,20}^{10} = 1$.

- One band can be re-used at multiple nodes
- Band 1 at nodes 7 and 16
The 20-Node Network: Flow Rates

- All sessions use multi-hop routing
- Multi-path routing is used for session 1

Session 1: \( f_{2,10}(1) = 103.30 \), \( f_{8,2}(1) = 103.30 \), \( f_{11,10}(1) = 15.86 \), \( f_{12,8}(1) = 103.30 \), \( f_{12,11}(1) = 15.86 \), \( f_{16,12}(1) = 119.16 \);
Session 2: \( f_{1,7}(2) = 13.24 \), \( f_{7,3}(2) = 13.24 \), \( f_{18,20}(2) = 13.24 \), \( f_{20,1}(2) = 13.24 \);
Session 3: \( f_{12,11}(3) = 52.96 \);
Session 4: \( f_{13,14}(4) = 39.72 \), \( f_{14,17}(4) = 39.72 \);
Session 5: \( f_{15,19}(5) = 26.48 \), \( f_{19,6}(5) = 26.48 \).
Routing Topology for A 30-Node Network
Routing Topology for A 50-Node Network

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Chapter 12 Summary

- Build mathematical models for each layer
  - Exploit properties of multi-hop CR networks

- Case study: Throughput maximization problem
  - Apply our models to formulate an optimization problem
  - Obtain near-optimal solution by branch-and-bound