Chapter 11

Information Theoretic Limits of Cognitive Radio Networks
Outline

- Introduction
  - Rise+importance of cognitive networks
  - Types of cognitive behavior
- Information theory basics
- Interference avoiding behavior
- Interference controlled behavior
- Interference mitigating behavior
- Summary
Introduction

- **Primary users:** primary spectrum license holders. Generally have priority access to spectrum.
- **Secondary users:** may access, under certain negotiated conditions, spectrum which is licensed out to primary users. Generally considered to use cognitive radio technology.
Introduction

Primary users

Secondary/cognitive users

“Cognitive Radio Communications and Networks: Principles and Practice”
Introduction

- In this chapter we focus on the information theoretic limits of communication in networks in which a subset of devices are cognitive radios.
- Cognitive radios have extra flexibility and capabilities: this must be modeled information theoretically!
- Different amounts of cognition are possible
Types of cognitive behavior

- How to manage interference is critical in wireless networks
- Cognitive behavior may be differentiated by how interference is handled:
  - Interference avoiding
  - Interference controlled
  - Interference mitigating
Types of cognitive behavior: interference avoiding

Secondary users communicate so as to completely avoid interfering with the primary users.

``white-space-filling''
``spectral gap filling''
``orthogonal''
Types of cognitive behavior: interference controlled

Secondary users communicate and control the amount of interference they cause the primary users.

``spectrum underlay``
``interference- temperature``
Types of cognitive behavior: interference mitigating

Secondary users communicate and mitigate the interference seen from the primary to the secondary receiver through exploitation of side information.

``cognition``

"Cognitive Radio Communications and Networks: Principles and Practice"
Information theory basics

x: channel inputs in alphabet $X$
y: channel outputs in alphabet $Y$

In information theory, a channel is modeled as a set of conditional distributions $p(y|x)$ for each $x$ in $X$. 
Information theory basics

Two most common channels:

- Discrete Memoryless Channel
- Additive White Gaussian Noise Channel
Information theory basics

Measures of interest:

1) Capacity/capacity regions: largest rate / rate regions at which reliable communication may be ensured.

2) Throughput scaling laws: how the sum throughput in a network scales with the number of nodes in the network.
Entropy and mutual information

Mutual information between random variables $X$ and $Y$:

$$I(X; Y) = \sum p(x, y) \log \left( \frac{p(x, y)}{p(x)p(y)} \right)$$

$$= H(X) - H(X|Y)$$

$$= H(Y) - H(Y|X)$$
Channel capacity

Information channel capacity:

\[ C = \max_p I(X; Y) \]

Operational channel capacity:

Highest rate (bits/channel use) that can communicate at reliably

Channel coding theorem says:

Information capacity = operational capacity

"Cognitive Radio Communications and Networks: Principles and Practice"
Gaussian channel capacity

\[ C = \frac{1}{2} \log \left( \frac{|h|^2 P + P_N}{P_N} \right) \]

\[ = \frac{1}{2} \log (1 + SNR) \text{ (bits/channel use)} \]

What about bits/second and bandwidth of the channel?

\[ C = W \log_2 \left( 1 + \frac{P}{W N_0} \right) \text{ (bits/second)} \]
Capacity regions

Point to point *capacity*

Multi-user *capacity region*

"Cognitive Radio Communications and Networks: Principles and Practice"
Scaling laws

• [Gupta+Kumar 2000]: Non-cooperative ad hoc networks
  - per-node throughput $\sim O(1/\sqrt{n})$
  - Degradation is due to multi-hop and **interference** between nodes

• [Ozgur, Leveque, Tse 2007]: Cooperative ad hoc networks
  - nodes may cooperate as in a MIMO system
  - per-node throughput $\sim O(1)$ (**constant**)
Classical multi-user channels

- Relay channel
- Multiple access channel
- Broadcast channel
- Interference channel
Relay channel

A source and destination communicate through the help of a relay.
Capacity is only known for certain classes of channels.
Cognitive nodes may act as relays for primary nodes.
Multiple-access channel

Two independent sources wish to communicate with the same destination.

The capacity region is well understood.

Superposition coding and successive decoding are key to achieving capacity.
Broadcast channel

A single transmitter wishes to communicate with two independent receivers.

The capacity region is unknown in general but is known for the Gaussian MIMO broadcast channel.

Dirty paper coding has been shown to be capacity-achieving.
Interference channel

Two independent transmitters wish to communicate with two independent receivers.

Capacity region is unknown in general, though is known to within a couple of bits in the Gaussian noise case.

Rate-splitting achieves the largest known rate-region.
Cognitive channels?

How can we model channels in which some of the nodes are *cognitive radios*?

Can we draw inspiration/techniques from the "classical" relay, multiple-access, broadcast and interference channels?
Cognitive channels

In the remainder, we will explain the following cognitive behaviors:
Cognitive channels

For the purpose of illustration we assume an AWGN channel model described by the equations

\[ Y_1 = X_1 + h_{21} X_2 + N_1, \quad N_1 \sim \mathcal{N}(0,1) \]
\[ Y_2 = h_{12} X_1 + X_2 + N_2, \quad N_2 \sim \mathcal{N}(0,1). \]
Interference-avoiding behavior

**Secondary user knowledge:** ideal knowledge of exactly when/where (time/freq/code) the primary user is transmitting.

**Secondary user behavior:** fills in the gaps whenever the primary user is **not** transmitting.

White-space filling region (a)

\[ \{(R_1, R_2) | 0 \leq R_1 \leq \eta C(P_1), 0 \leq R_2 \leq (1 - \eta) C(P_2), 0 \leq \eta \leq 1 \} \]
Interference-controlled behavior

**Secondary user knowledge**: the interference conditions at the primary Rx and the channel between secondary Tx and primary Rx

**Secondary user behavior**: transmits with power such that the received interference at the primary Rx is within pre-defined threshold

\[
\text{Simultaneous-transmission rate region (b)} = \left\{(R_1, R_2)\left| 0 \leq R_1 \leq C \left(\frac{P_1}{h_{21}^2 P_2^* + 1}\right), \right. \right. \\
\left. 0 \leq R_2 \leq C \left(\frac{P_2^*}{h_{12}^2 P_1 + 1}\right), \ 0 \leq P_2^* \leq P_2 \right\}.
\]
Interference-mitigating behavior

Type 1:

**Secondary user knowledge:** the codebooks of the primary user

**Secondary user behavior:** the secondary Rx may be able to decode the primary message and strip it off the received signal
Interference-mitigating behavior

Type 2:

**Secondary Tx knowledge:** the codebooks of the primary user and the message of the primary user

**Secondary user behavior:** the secondary Tx codes so as to relay the primary message as well as mitigate its interference at the secondary Rx using *dirty-paper coding*
Aside: dirty-paper coding

In the following channel model, if interference S is known non-causally at the Tx but not the Rx, the capacity region is that of an interference-free system!

\[ Y = X + S + N, \quad E[|X|^2] \leq P, \quad N \sim \mathcal{N}(0, Q) \]

\[
C = \max_{p(u|s)p(x|u,s)} I(U; Y) - I(U; S)
\]

\[
= \frac{1}{2} \log_2 \left( 1 + \frac{P}{Q} \right).
\]
Behaviors

Interfere-mitigating cognitive behavior (type 2) fits well within the context of competitive, cooperative and cognitive behaviors:

(a) Competitive

(b) Cognitive

(c) Cooperative

“Cognitive Radio Communications and Networks: Principles and Practice”
Interference-mitigating behavior

Asymmetric cooperation rate region (d)

\[
\{(R_1, R_2) | 0 \leq R_1 \leq C \left( \frac{(\sqrt{P_1} + h_{12}\sqrt{\psi}P_2)^2}{h_{12}^2(1-\psi)P_2 + 1} \right),
  0 \leq R_2 \leq C ((1-\psi)P_2), 0 \leq \psi \leq 1 \}
\]
Chapter 11 Summary

The achievable rate regions depend on the amount of side-information the cognitive Tx/Rx has about the primary transmission.

Studied 3 different types of behavior: interference avoiding, interference-controlled and interference-mitigating cognitive behavior from an information theoretic perspective.